Connecting Mathematics Education Research to Practice

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The disjunctures that exist between research and practice and ways to bridge them are examined. An underlying theme of the presentation is a consideration of the roles played by beliefs and values in decision making about school mathematics, and the conflicts that can exist due to unexamined opposing beliefs and values.

> Ultimately, the two separate themes of designing learning experiences that meet the needs of students, and understanding more deeply what is involved in the way humans think about mathematics, may indeed be seen as intimately related. Indeed, it is hard to see how those involved in either enterprise can make an optimal contribution if they do not become allies and learn to work together in the closest possible way. (Davis, 1996, p. 298)

As researchers, we care about mathematics learning, teaching, curriculum, and assessment in school mathematics. We choose research topics because we want to know more about why a particular concept is so difficult to learn, why teachers differ so in their instructional effectiveness, what curriculum best serves our students, and how assessment can be aligned with and even drive the curriculum. Yet we all know that much of our research does not get translated into practice. I would like to explore with you some of the reasons for this lack of connection, and possible ways to change this situation.

Mary Kennedy, in an Educational Research (1997) article, discussed what she called the "apparent failure" of research to influence teaching. She hypothesised four reasons for the disjuncture between research and practice:

1. The research itself is not sufficiently *persuasive or authoritative*;
2. The research has not been *relevant* to practice. It has not been sufficiently practical, it has not addressed teachers' questions, nor has it adequately acknowledged their constraints.
3. Ideas from research have not been *accessible to teachers*. Findings have not been expressed in ways that are comprehensible to teachers.
4. The education system itself is intractable and unable to change, or it is conversely inherently unstable, overly susceptive to fads, and consequently unable to engage in systematic change. (p. 4; italics hers)

These points provide a framework for reflection on our research and why it is so frequently disconnected from what happens in classrooms throughout your country and mine. I use her first three points to structure this presentation.

Our Research Must Be Sufficiently Persuasive and Authoritative Before It Can Affect Practice

This point naturally leads us to consider the audience for our research reports. Who do we want to persuade? Here are four audiences.

Teachers. I personally know teachers who are hungry for research that is relevant and accessible, but I also know that there are many mathematics teachers who have little regard for mathematics education research. For many teachers, research is not persuasive because it is viewed as irrelevant to their professional lives. Can this situation change? I will return to this point.

Policy makers. Experimental research has made a strong comeback in the U.S., in all educational areas. If the research is not experimental, it is simply not considered to be research, and is therefore not considered when making policy decisions. I don't know what the situation is here, but I suspect that your policy makers are seldom basing their decisions on the research reports you write. Can this situation change?

Mathematicians. I would like to talk about this audience with a story. Under the auspices of the Conference Board of Mathematical Sciences, a group of writers (a mathematician and three mathematics educators), with the assistance of a large advisory board consisting of mathematicians and mathematics educators, produced a document on the preparation of teachers of mathematics. The document is intended for the mathematics community, and by the time of this MERGA conference it should be in print.(A draft has been on the MAA website for some time and many mathematicians have reacted to it.) In the initial outline of this document, there was to be a chapter on how people learn mathematics. In my role as one of the authors, I offered to take a first stab at the chapter, to be used by others as a springboard from which to develop ideas for this chapter. It was suggested to me by mathematicians that I should not base this chapter on research in mathematics education, because that research was not respected by many mathematicians. Soon after, this chapter was deleted from the outline for the book. I have heard similar stories from others, and I suspect that there are similar stories some of you could tell. Most mathematicians are not persuaded that our research has any authority. Can this situation change?

Parents. Successful parents want their children to succeed. Many of them believe that what worked for them will work for their children. Can this situation be changed?

Thus there are important audiences—mathematicians, teachers, policy makers and parents—who are not, for the most part, persuaded by our research, and who question the authoritativeness of what we have to share with them. The questions of what counts as authoritative and persuasive research are sticky ones. These questions were discussed in invited forum pieces in the March 1999 issue of JRME, but worded in a slightly different manner. The question asked of these authors was: What counts as evidence? In the response by Carnine and Gersten, they claimed that the

> "phenomenal body of research" of the past decade has clarified the "variables and issues that are critical for students and teachers", and that "We now need to build upon this base using rigorous controlled studies. … We need not only to conduct more experimental studies but also to ensure they are of the highest quality." (p. 142)

Is this an answer to the question of what counts as evidence? Carnine, a behavioral psychologist, was funded by the California State Board of Education to review the literature in mathematics education and produce a document to be used in making decisions regarding school mathematics. He located about 9000 studies, then tossed out all but about 1000 studies because they were not experimental, but less than 100 made it through the evaluation criteria he had developed. This review was sent to every school principal in

California with instructions that it be used when making decisions on school mathematics. Can all of our research be reduced to less than 100 "worthwhile" studies?

Many claim that it is not possible to conduct rigorous experimental studies to answer some of the questions important to many audiences, questions such as whether or not the U. S. curricula developed in response to the 1989 NCTM Standards are better or worse than "the" traditional mathematics curriculum. This question is quite important because parents do not want "untested" curricula used in schools, nor do they want their children used as "guinea pigs" in studies of curricula.

The Carnine and Gerston article was not the only forum piece requested. In the second forum piece, Lester and Wiliam said

> The relation between knowledge claims and evidence … is determined, in large part, by the set of beliefs, values, and perspectives operating in the context in which the empirical data are being assessed. How researchers go about convincing others of the claims they make and how they defend their claims on ethical and practical grounds are, only in part, matters of marshaling adequate, contextualized evidence embedded in sets of beliefs and theories. Indeed, convincing others is also a matter of persuading them to accept the values the researcher hold about the objects and phenomena being studied as well as about the very purpose of the research itself. (p. 136)

This quote brings me to the heart of what I want to consider today: the role of beliefs and values in decision making about school mathematics. Schwartz tells us that "Values function as the 'criteria people use to select and justify actions and to evaluate people and events.'" (1992, p. 1) If we return to the question of "which curriculum" to use, I do not believe that question can be answered through comparison studies. Rather, we should choose a curriculum because it has been shown to develop concepts and skills we value. For one set of teachers, administrators, and parents, this might mean a strong skill-oriented curriculum, for another set of teachers, administrators, and parents, this might mean a focus on problem solving and sense-making. There are very different values at work in this choice. How many teachers, administrators, and parents have examined their own values and the implications of those values, and are consciously aware of the role their values play in their decisions about aspects of schooling?

In education, as in other important aspects of our lives, values and beliefs are firmly held and often jealously guarded. When they conflict with the beliefs and values of others, we become adversaries unless each side can come to understand and respect the values and beliefs of the "other" side. Here are three areas in which there are strong conflicting beliefs and values about school mathematics.

What Is Mathematics? Or at Least, What Do We Believe It Is?

The prevailing public view of mathematics is that it is a set of rules, skills, and facts that need to be learned, maintained, and drawn upon when necessary. Others view mathematics as a unified (but static) body of knowledge, with all the parts logically connected, there to be discovered by humans. Holders of both of these views tend to want a curriculum that assists students in learning the rules. They believe that the mathematics ought to be carefully and logically sequenced so that children come to learn, bit by bit, the parts of this body of knowledge appropriate for their grade level. Mathematicians are seen to have the power to discover new facets of mathematics, but even getting a glimpse of this power is not considered an appropriate goal for school mathematics. Others view mathematics as dynamic—invented rather than discovered. When the mathematician Rueben Hersh was asked, "What is mathematics?" he responded that "Mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas" (1986, p. 22). He went on to say that mathematics is invented and created by humans in response to needs of science or daily life, and that "*knowing* mathematics is *making* mathematics". My late colleague Alba Thompson found this view of mathematics reflected in many of the documents of the last decade or so that have advocated reforming the curriculum.

> The conception of mathematics teaching that can be gleaned from these documents is one in which students engage in purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation. This view of mathematics teaching is in sharp contrast to alternative views in which the mastery of concepts and procedures is the ultimate goal of instruction. However, it does not deny the value and place of concepts and procedures in the mathematics curriculum. (Thompson, 1992, p. 128)

How Do Children Best Learn Mathematics?

For several decades now there have been competing views among psychologists about how children learn. The behaviorist tradition is based on stimulus-response theory, and teaching that reflects this theory is usually quite directive. Skill learning is paramount, and skills are thought to be acquired through a process of practice of isolated skills, finally coming together to form more complex skills. This view of learning mathematics can be found in the mathematics standards in my own state. When they were approved by the California State Board of Education, an editorial in my own local newspaper, the San Diego Union Tribune, contained this statement.

> The tentative accord on math standards is especially encouraging because it underscores the importance of learning the basics. Ralph Cohen, a Stanford math professor, who helped draft the tougher standards, put it best when he said teachers should not expect children to understand concepts without first gaining a solid grounding in the fundamentals.

Yet there are others who believe that skills should not be taught separated from conceptual understanding. I include myself in this group, and I suspect that many of you are also in this group. I consider teaching to be a process of helping children organize their knowledge through making many connections and forming relationships. I view knowledge as a connected web in "which the linking relationships are as prominent as the discrete pieces of information." (Hiebert & Lefevre, 1986, p. 3-4) The question of whether developing skills with symbols leads to conceptual understanding, or whether the presence of basic understanding should precede symbolic representation and skill practice, is one of the basic disagreements that undermines our ability to develop and adopt agreed-upon curricula and teaching practices.

What Do We Mean by "Doing Well" in Mathematics?

We would all say that we want our children to do well, to gain expertise in mathematics. But what is expertise? Giyoo Hatano (1988), a Japanese psychologist, distinguished between two types of expertise. The first he called *routine* expertise. A person with routine expertise can solve routine problems quickly and accurately, using automatised procedures. Such expertise is acquired when one repeatedly solves problems of the same type, for which efficiency and right answers are valued but for which understanding is not so important. This type of expertise is sufficient for many situations in which mathematics is used. It is this type of expertise, I believe, that many parents wish for their children, because it is the type of expertise they have. The acquisition of routine expertise in carrying out paper-and-pencil calculations has long been an objective of schooling in mathematics, and certainly, before calculators became ubiquitous, routine expertise was a necessity. But there is another type of expertise described by Hatano. He called this *adaptive* expertise. This type of expertise requires understanding of how and why and what procedures work, and how these procedures can be modified to fit changing constraints of a problem. It is this adaptive expertise that those who want to reform the curriculum believe should be our goal. Hatano is not alone in his thinking about expertise. Studies of expertise in different cognitive domains led Dreyfus and Dreyfus (1986) to hypothesise that practice of rules does not necessarily develop true expertise, which in fact includes many capabilities that are acquired independently of rules. So—what kind of learning do we want for our children? What do we value? Related to these questions, Hiebert (1999) said

> We now know that we *can* design curriculum and pedagogy to help students meet the ambitious learning goals outlined by the NCTM Standards. The question is whether we value those goals enough to invest in opportunities for teachers to learn to teach in the ways they require. (p. 16)

This statement can speak to standards produced here as well as in my country. I think we need to take more seriously the need for research on policy issues related to mathematics learning. Rather than trying, for example, to show parents how good something is, or even worse, to leave them out of the loop, we need to find, or perhaps invent, ways to provide opportunities that will allow policy makers and parents to examine their values, to clarify for themselves what learning goals they have for their children, and then to come to the table ready to talk about how these goals can be achieved. *Research can be persuasive and authoritative only to the extent that people are fully aware of what they value and why, and are open to considering evidence that supports or refutes those values.*

Our Research Must be Relevant to Practice

Ed Silver, in a 1990 NCTM Yearbook chapter, spoke of three ways in which research can affect teaching practices. I will discuss each of these components in the context of *Journal for Research in Mathematics Education (JRME)* articles I accepted as editor, although of course many other examples could be provided in each case.

First, many research *findings* can be applied to school settings. Of course, many research studies are limited in scope and do not have major implications in and of themselves, but most studies are situated in a body of literature and extend, or perhaps refute, what came before. Thus research results can be discussed in terms of what is already known and how these results fit within a body of research that is applicable to practice. For example, the very successful use by the Dutch (Klein, Beishuizen, & Treffers, 1998) of the "empty number line" as a didactic learning aid to model addition and subtraction in second grade is but one study in a Dutch program of research aimed at producing flexible mental addition and subtraction.

Some research reports stand on their own in terms of the powerful messages they provide. Boaler (1998) spent three years gathering case-study data in two secondary English schools with very different approaches to teaching mathematics. In her words, “Students who followed a traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations.” (p. 41)

In another study, Linchevski and Kutscher (1998), described the effects of heterogeneous grouping and homogeneous grouping on student achievement. They found that high ability students were not harmed by heterogeneous grouping, but middle and low ability students *were* harmed by homogeneous grouping. The study took place over two years, Grades 7 and 8, and it was carefully designed and undertaken. The study provides a great deal of food for thought regarding our practices of tracking students, and it perhaps provides ideas for experimentation by teachers and principals at all levels.

Another way that research can inform practice, according to Silver, is through the methods used, particularly the tasks designed for the research study. Researchers spend a lot of time developing the tasks they use. Designing tasks that can produce measurable differences in the outcome of a study is not a trivial matter. MacGregor and Price (1999) spent a great deal of time, I am sure, designing and piloting the items they used to measure constructs important to their study of the relation of language proficiency to algebra learning. These tasks can be used, or even modified, by others interested in this question. I could name many research studies from which I have myself taken the tasks and transported them into my own teaching. But too often we look only for results from research and we overlook the small gold mines along the way.

A third way in which research can have applications to a school setting is in the borrowing of theoretical constructs and perspectives of researchers. Over 20 years ago Jeremy Kilpatrick said that

> Too many mathematics educators have the wrong idea about research. They give most of their attention to the results … In a nontrivial sense, however, the results are the least important aspect of a research study … the most important aspect of a research study is the constructs and theories used to interpret the data. A landmark research study is one that confronts us with data analyzed and organized so as to shake our preconceptions and force us to consider new conceptions. (Kilpatrick, 1981, p. 27)

I would argue that the studies by Boaler and by Linchevski and Kutscher are landmark studies. In both cases the findings are not simply the results, but it is the manner in which the data from these studies were compiled and analysed that is so compelling. How would we expect these studies to influence practice? How can these studies speak to teachers and administrators? What can we do to help teachers and administrators have access to these and other studies that have important messages for mathematics schooling?

There are of course other examples of the constructs and theories of researchers being relevant to teachers. In Adler’s (1999) study, she found that when a teacher attends too closely to the mathematical language used by students, in this case in a South African bilingual classroom, the mathematics itself moved outside the focus of the students. Where is the appropriate balance between focusing on the mathematics and on the language used to communicate about it? Adler used the construct of "transparency" to discuss the ways in which talk needs to be both visible and invisible so that students can access the mathematics. In another article, Jacobson and Lehrer (2000) spoke of the ability of second grade teachers to "appropriate" a video on quilting, focusing on geometric transformations, as dependent on the teachers' knowledge of student thinking about space and geometry. The idea of teachers' ability to "appropriate", to take ownership of a curriculum unit being dependent on their knowledge of student thinking, can certainly be reflected upon and discussed outside the mathematics of this particular study.

The Silver chapter provides one structure for thinking about the relevance of research. But there are other ways to think about relevance. Here is one suggested by Tom Kieren. He considered the research on the impact of the environment on learning and of interactions in it on the mathematical thinking of students:

> Because much of such research now takes place *in vivo* in classrooms or other educational environments instead of *in vitro* (under specially controlled conditions) its potential direct relevance to practice is raised. The unit of analysis now becomes the teacher/student/environment and the actions and thinking of the student at once act to bring forth a world of mathematical significance and are occasioned by the possibilities and interactions in that world … These research practices illuminate how research might relate to the practices of teaching and learning. Such practices point to alternative effective teaching practices and to new emphases:
>
> • on listening to rather than simply listening for;
> • on acting with students in doing mathematics rather than simply showing student how to do mathematics;
> • on establishing effective discourses of mathematical argument or mathematical conversation rather than simply the discourse of telling, interrogating, and evaluating;
> • on the mechanisms of students' mathematical thinking rather than simply on student' answers;
> • on the teacher and students as fully implicated by their actions each in the learning of the other; and
> • on the teacher as co-developer of a lived mathematics curriculum not just a recipient of or a conduit for a predecided curriculum. (Kieren, 1997, pp. 32-33)

Ideas from Research Must be Accessible to Teachers

Unfortunately, if the research published in research journals were written to be easily accessible to teachers, the articles probably would not pass muster as rigorous research reports. Cronbach and Suppes said in 1969 that research is disciplined inquiry, that it is "inquiry conducted and reported in such a way that the argument can be painstakingly examined." (p. 15) This description of research reporting has not really changed. The down side is, of course, that the requirement of preparing a research report for painstaking examination by peers often renders the report dense and difficult to read and understand by the very people who can most benefit from the research and implement its findings—teachers and policy makers. There needs to be some bridge constructed between research reports and the type of reports that allow authors to communicate with teachers and policy makers.

> The problem of accessibility is not merely one of placing research knowledge within *physical* reach of teachers, but rather one of placing research knowledge within the *conceptual* reach of teachers, for if research encouraged teachers to reconsider their prior assumptions, it might ultimately pave the way for change. (Kennedy, 1997, p. 7)

I would add to this quote that the research needs to be within the *practical* reach of teachers. Teachers have neither the time nor the inclination to deal with too much detail, with statistical analysis, or with long literature reviews. I encourage researchers to take upon themselves the responsibility of finding ways to share research with teachers, whether it by through collaborative projects, or writing for teachers and submitting to journals for teachers or other ways of placing research within physical, conceptual and practical reach of teachers.

Conclusion

The theme of this conference is "Numeracy and Beyond". All of our research audiences have very strong beliefs about numeracy—often limited to what they think children need to know about arithmetic operations. Unfortunately, those beliefs are aligned with what they themselves learned, and what they learned is not adequate for their children's needs even today, let along tomorrow. To go beyond present day concepts of numeracy will require that we in some way cause shifts in what the public believes and values in the arena of numeracy.

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